Energy and Energy Conservation

Reference Guide

Equipment Setup

Roller Coaster. I

Investigation Guides

A-1 Speed on the Roller Coaster 3

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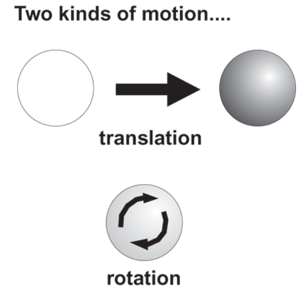
C-2 Rotational Kinetic Energy

Key Question: How can you calculate the rolling energy of a marble?

A marble on the roller coaster uses two kinds of motion to get from the beginning to the end of the track. *Translational motion* is the forward or linear motion of the marble. Thus far in working with the roller coaster, our calculations and descriptions of the marble's motion have only considered translational motion. The rolling of the marble along the track is the second kind of motion that the marble uses. In this Investigation, students will learn to measure and calculate the speed and kinetic energy of an object that has *rotational motion*.



Preparation

Students should have a strong background in working with and manipulating algebraic equations. Students should also be familiar with the motion equations and terms for describing linear (translational) motion. A review prior to doing this Investigation is necessary for students to derive the most benefit from the experience.

Setup and Materials

Students work in groups of four at tables.

Each group should have:

- One physics stand
- One roller coaster
- One CPO timer and one photogate (with a 9-volt battery or AC adapter and cords for connecting the photogates)
- A metric ruler (at least 30 centimeters) or meter stick

Each student should have:

- A simple calculator
- Data from Table 2 and the *Energy versus distance* graph from *B-2 Conservation of Energy*
- Copy of the Investigation and answer sheet

The Investigation

Time

One class period

Leading Questions

- What is rotational kinetic energy?
- How do you calculate total kinetic energy of a rolling marble?

Learning Goals

In this Investigation, students will:

- Derive an equation for the rotational kinetic energy of a marble.
- Calculate the translational and rotational kinetic energies of a rolling marble.
- Compare the potential energy of the marble to the sum of its kinetic energies.
- Make and interpret a graph of the energy of the marble.

Key Vocabulary

translational speed, angular speed, kinetic energy, rotational kinetic energy, moment of inertia

1

1a. Equations are:

$$E_{Kr} = \frac{1}{2} I\omega^2$$

$$E_{Kr} = \frac{1}{2} I\left(\frac{v^2}{r^2}\right)$$

$$E_{Kr} = \frac{1}{2}\left(\frac{2}{5}mr^2\right)\left(\frac{v^2}{r^2}\right)$$

$$E_{Kr} = \frac{1}{2} mv^2$$

1b. Equations are:

$$E_P = mgh$$

$$E_{Kt} = \frac{1}{2} mv^2$$

1c. Equations are:

$$\Delta E_P = E_{Kt} + E_{Kr}$$

$$mg\Delta h = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

$$v^2 = \frac{10}{7}(g\Delta h)$$

$$v = \sqrt{\frac{10}{7}g\Delta h} = \sqrt{\frac{10}{7}g\left(h_i - h\right)}$$

C-2 Rotational Kinetic Energy

C-2

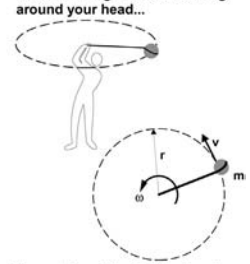
Question: How can you calculate the rolling energy of a marble?

In this Investigation, you will:

1. Derive an equation for the rotational kinetic energy of a marble.
2. Calculate the translational and rotational kinetic energies of a rolling marble.
3. Compare the potential energy of the marble to the sum of its kinetic energies.
4. Make and interpret a graph of the energy of the marble.

A marble on the roller coaster uses two kinds of motion to get from the beginning to the end of the track. *Translational motion* is the forward or linear motion of the marble. Thus far in working with the roller coaster, your calculations and descriptions of the marble's motion have only considered translational motion. The rolling of the marble along the track is the second kind of motion that the marble uses. In this Investigation, you will learn to measure and calculate the speed and kinetic energy of an object that has *rotational motion*.

When whirling a ball on a string around your head...



The motion of the ball is circular. The translational speed of the ball is related to the radius and angular speed of the circular motion.

1 Getting started

Describing the motion of an object often includes providing the speed of the object. When describing and measuring translational motion, speed is represented by the variable, *v*. The speed of an object that has rotational motion is called *angular speed* and is represented with the variable, ω (omega).

The following equation relates the angular and linear (translational) speeds of a sphere of radius *r* rolling on one edge.

Translational speed = Angular speed × the radius of the rolling object

$$v = \omega r$$

The kinetic energy (E_{Kr}) of a rigid, rolling object can be calculated using the angular speed (ω) and the *moment of inertia* (*I*) of the object.

$$E_{Kr} = \frac{1}{2} I\omega^2$$

The moment of inertia of an object is the term that describes the mass of a rotating object. Moment of inertia (*I*) is equal to two-fifths times the mass (*m*) of the object times the square of the distance (*r*). This mass is from the axis of rotation. See the diagram on the next page.

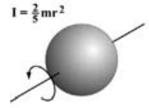
C-2

a. Use the equations on the previous page and the moment of inertia for a sphere to derive an equation for the rotational kinetic energy (E_{Kr}) in terms of mass (m) and the linear speed (v). This derived equation allows you to calculate rotational kinetic energy when you only know mass and linear speed.

b. Write down the equations for potential energy and translational (linear) kinetic energy in Table 1.

c. Using the energy equations (E_{Kr}, E_P, and E_{Kt}) and the law of conservation of energy, derive a new equation that predicts the speed of the marble from the acceleration of gravity, the initial height of the marble on the track, and the height at any later time.

$I = \frac{2}{5}mr^2$



sphere of radius r rotated about a major diameter

Table 1: Potential and Kinetic Energy Equations

Potential Energy (E_P)	Translational Kinetic Energy (E_{Kt})

2 Comparing predicted and calculated speeds

a. Fill in the first four columns in Table 2 below with the data from *C-1 Motion on the Roller Coaster* (Table 2 in Part 6). Do not fill in the predicted speed values from *C-1*.

b. Using the equation from Part 1, question c, calculate predicted speed. Record these values in Table 2.

c. Compare the predicted speed values you just calculated and the predicted speeds from Investigation *C-1 Motion on the Roller Coaster*. Which predicted values might be more accurate? Why?

Table 2: Comparing Predicted and Calculated Speed

Position on roller coaster (m)	Height (m)	Time from photogate A (sec)	Measured speed (m/sec)	Predicted speed (m/sec)

2

2a and 2b.

Table 2 data:

Position (m)	Ht (m)	Time from photogate A (sec)	Measured speed (m/sec)	Predicted speed (m/sec)
0.05	0.347	0.0452	0.42	0.529
0.15	0.293	0.0188	1.011	0.869
0.25	0.226	0.0133	1.429	1.302
0.35	0.166	0.0113	1.681	1.592
0.45	0.140	0.0108	1.759	1.702
0.55	0.172	0.0116	1.638	1.565
0.65	0.241	0.0164	1.159	1.218
0.75	0.297	0.0254	0.748	0.837
0.85	0.308	0.0302	0.629	0.739
0.95	0.270	0.0192	0.99	1.0383
1.05	0.206	0.0133	1.429	1.405
1.15	0.137	0.0112	1.696	1.715

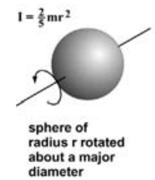
2c. The table below shows the percent differences between the measured and predicted speeds for Investigation C-1 and this Investigation which included rotational motion in calculating predicted speed. With the exception of the beginning of the track, the percent differences are much lower when rotational motion is included in the prediction. These predicted values are more accurate than the ones made in Investigation C-1. The results indicate that rotational motion is a significant component of the marble's motion and should be considered when predicting of the marble's speed.

Position (m)	Translational motion only % different between measured and predicted speeds (m/sec)	Translational + Rotational motion % difference between measured and predicted speeds (m/sec)
0.05	49	26
0.15	2	14
0.25	8	9
0.35	12	5
0.45	15	3
0.55	13	5
0.65	24	5
0.75	32	12
0.85	39	18
0.95	24	5
1.05	16	2
1.15	20	1

a. Use the equations on the previous page and the moment of inertia for a sphere to derive an equation for the rotational kinetic energy (E_{Kr}) in terms of mass (m) and the linear speed (v). This derived equation allows you to calculate rotational kinetic energy when you only know mass and linear speed.

b. Write down the equations for potential energy and translational (linear) kinetic energy in Table 1.

c. Using the energy equations (E_{Kr}, E_P, and E_{Kt}) and the law of conservation of energy, derive a new equation that predicts the speed of the marble from the acceleration of gravity, the initial height of the marble on the track, and the height at any later time.



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a. Fill in the first four columns in Table 2 below with the data from *C-1 Motion on the Roller Coaster* (Table 2 in Part 6). Do not fill in the predicted speed values from *C-1*.

b. Using the equation from Part 1, question c, calculate predicted speed. Record these values in Table 2.

c. Compare the predicted speed values you just calculated and the predicted speeds from Investigation *C-1 Motion on the Roller Coaster*. Which predicted values might be more accurate? Why?

Table 2: Comparing Predicted and Calculated Speed

Position on roller coaster (m)	Height (m)	Time from photogate A (sec)	Measured speed (m/sec)	Predicted speed (m/sec)

C-2

3 Graphing the data

The predicted speed values in Table 2 take into consideration that the marble has rotational motion. Now, make a graph that shows the relationship between the measured speeds and these predicted speed values. Place predicted speed on the *x*-axis and measured speed on the *y*-axis.

4 Analyzing the data

a. What can you tell from your graph? Write down one to two sentences that describe how the measured speed compares to the theoretical prediction. Your answer should contain a numerical estimate of the differences between the two values expressed as a percentage. Were the values more similar in this Investigation than they were in *C-1 Motion on the Roller Coaster*?

b. Is there something that is still not included in the theory that might explain why the predicted speed values do not exactly match the measured values?

5 Comparing energies

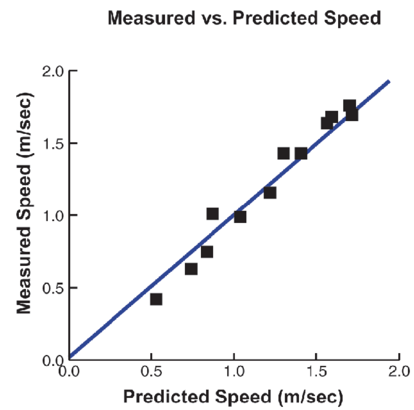
Using your new equation from Part 1, question c, calculate total kinetic energy (translation plus rotation). Then, calculate the potential energy and total energy (potential plus total kinetic). Record your calculations in Table 3.

Table 3: Potential, Total Kinetic, and Total Energy

Position on roller coaster (m)	Height (m)	Speed (m/sec)	Potential energy (joules)	Total kinetic energy (joules)	Total energy (joules)

3

Graph:



4

4a. The predictions made taking rotational motion into account more closely resemble the measured values. The percent deviations are less than 10% for 8 out of 12 positions. This is better than the predictions made in C-1 which did not consider rotational motion.

Position (m)	% difference
0.00	29.4
0.10	1.2
0.20	2.3
0.30	2.6
0.40	3.0
0.50	6.4
0.60	12.0
0.70	42.2
0.80	85.7
0.90	37.7
1.00	13.5
1.10	9.2
1.20	7.5
1.30	8.0

4b. Friction is a significant factor. Note that it has its greatest effect at low speeds.

5

Table 3 data:

Pos (m)	Ht (m)	Meas. speed (m/sec)	PE (joules)	Total KE (joules)	Total energy (joules)
0.00	0.361	0.17	0.0998	0.0006	0.1004
0.10	0.313	0.84	0.0865	0.0141	0.1006
0.20	0.245	1.31	0.0677	0.0339	0.1016
0.30	0.180	1.56	0.0497	0.0479	0.0976
0.40	0.150	1.67	0.0415	0.0548	0.0963
0.50	0.163	1.57	0.0450	0.0487	0.0937
0.60	0.223	1.25	0.0616	0.0308	0.0925
0.70	0.289	0.71	0.0799	0.0100	0.0899
0.80	0.319	0.42	0.0882	0.0035	0.0917
0.90	0.297	0.69	0.0821	0.0094	0.0914
1.00	0.233	1.18	0.0644	0.0275	0.0919
1.10	0.165	1.52	0.0456	0.0456	0.0912
1.20	0.115	1.73	0.0318	0.0589	0.0907
1.30	0.105	1.76	0.0290	0.0611	0.0901

C-2

3 Graphing the data

The predicted speed values in Table 2 take into consideration that the marble has rotational motion. Now, make a graph that shows the relationship between the measured speeds and these predicted speed values. Place predicted speed on the *x*-axis and measured speed on the *y*-axis.

4 Analyzing the data

a. What can you tell from your graph? Write down one to two sentences that describe how the measured speed compares to the theoretical prediction. Your answer should contain a numerical estimate of the differences between the two values expressed as a percentage. Were the values more similar in this Investigation than they were in *C-1 Motion on the Roller Coaster*?

b. Is there something that is still not included in the theory that might explain why the predicted speed values do not exactly match the measured values?

5 Comparing energies

Using your new equation from Part 1, question c, calculate total kinetic energy (translation plus rotation). Then, calculate the potential energy and total energy (potential plus total kinetic). Record your calculations in Table 3.

Table 3: Potential, Total Kinetic, and Total Energy

Position on roller coaster (m)	Height (m)	Speed (m/sec)	Potential energy (joules)	Total kinetic energy (joules)	Total energy (joules)

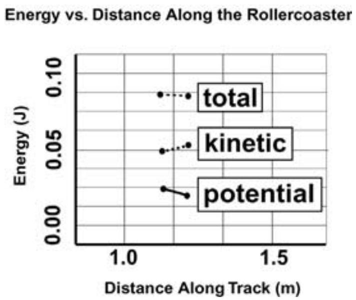
3

6 Graphing energy data

C-2

Use your energy data to plot a graph showing three parameters—potential energy, kinetic energy, and total energy—plotted against distance along the roller coaster. Use a solid line for potential energy, a dotted line for kinetic energy, and a dashed line for total energy. Also, make a legend indicating what each line represents.

The example shows what this graph might look like for the first few points:



7 Analyzing the data

a. From your graph, what can you say about the energy of the marble? Write down one to two sentences that describe the different types of energy the marble has at different places along the roller coaster.

b. What percentage of the marble's energy is left at the end of the roller coaster just before it stops (or at the last place you measured)?

c. What happens to the lost energy of the marble?

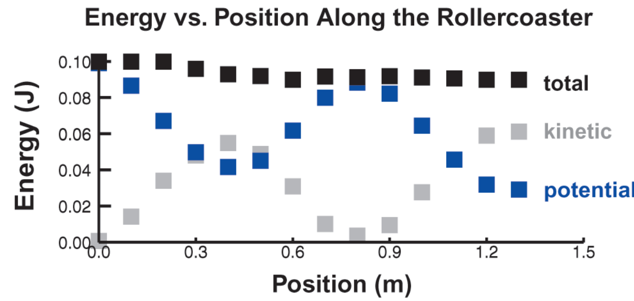
8 Comparing results

In this Investigation, you determined the total kinetic energy of the marble by adding the translational and rotational kinetic energies. Your handout includes comparable data, but rotational kinetic energy was not included in the calculations for this data set.

Write a paragraph that compares your data and graph made using total kinetic energy with the data and graph your teacher gave you. In your paragraph, state the advantages of including total kinetic energy (translational plus rotational) and the advantages of only including translational in your calculations. In other words, is it always necessary to include rotational kinetic energy in your calculations? Explain your responses to these questions.

6

Graph:



7

7a. The graph shows that the overall total energy of the marble decreases as it moves along the track. The graph also shows that as potential energy decreases, kinetic energy increases and vice versa.

7b. Since the marble started with 0.1004 joules and finished with 0.0901 joules, it has lost 0.0103 joules, or 10.3%. Therefore, there is 89.7% left.

7c. The energy is dissipated as heat and the friction that wears away portions of the track.

8

Note: The data used in the handout is from Investigation B-2.

Both data sets (B-2 and C-2) illustrate that (1) the total energy of the marble appears to decrease along the track, (2) the potential energy plot follows the general shape of the roller coaster height, and (3) the kinetic energy plot is the inverse of the potential energy plot. When you take rotational kinetic energy into consideration, you account for more kinetic energy in the system (~90% versus 81%). In other words, including rotational kinetic energy gives you a more accurate picture of how much energy is transformed into heat and how much is transformed into kinetic energy.

1. rotational inertia or moment of inertia
2. friction
3. The marble at 28 cm has more potential energy than the marble at 18 cm. If the marbles are in motion, and part of the same system, the marble at 18 cm has more kinetic energy.
4. Answer is:

$$\left(mgh_1 + \frac{7}{10}mv^2\right) = \left(mgh_2 + \frac{7}{10}mv^2\right)$$

Mass factors out on both sides

leaving speeds and heights as unknowns:

$h_1 = 0.18\text{cm}$ $\quad v_1 = ?$

$h_2 = 28\text{cm}$ $\quad v_2 = (0.019\text{m} / 0.0455\,\text{sec}) = 0.418\text{m/sec}$

Solve equations for v_1 :

$$v_1 = \sqrt{\left[\frac{10}{7}g\left(h_2 - h_1\right) + \left(v_2\right)^2\right]} = 1.25\text{m/sec}$$

5. The solid wheels have less rotational inertia than the spoked wheels and accelerate more easily than the spoked wheels.
6. Answer is:

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = 11.3\text{m/sec}$$

Curriculum Resource Guide: Rollercoaster

Credits

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Curriculum Resource Guide: Rollercoaster

ISBN 1-58892-049-6
2 3 4 5 6 7 8 9 - QWE - 05 04 03

Cambridge Physics Outlet
26 Howley Street,
Peabody, MA 01960
(800) 932-5227
http://www.cpo.com

Printed and Bound in the United States of America

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